Causality, Measurement, and Elementary Interactions

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Abstract

Signal causality, the prohibition of superluminal information transmission, is the fundamental property shared by quantum measurement theory and relativity, and it is the key to understanding the connection between nonlocal measurement effects and elementary interactions. To prevent those effects from transmitting information between the generating and observing process, they must be induced by the kinds of entangling interactions that constitute measurements, as implied in the Projection Postulate. They must also be non-deterministic as reflected in the Born Probability Rule. The nondeterminism of entanglement-generating processes explains why the relevant types of information cannot be instantiated in elementary systems, and why the sequencing of nonlocal effects is, in principle, unobservable. This perspective suggests a simple hypothesis about nonlocal transfers of amplitude during entangling interactions, which yields straightforward experimental consequences.

1 Introduction

Measurements are carefully designed arrangements of elementary, correlating interactions, but standard formulations of quantum theory treat the act of measurement as irreducible. This explanatory gap between microphysical evolution and macroscopic outcomes is a serious problem for the conventional theory of measurement. However, it should not lead us to overlook some particularly elegant features of that theory, and the extent to which it permeates and shapes fundamental physics.

The central place of the measurement postulates in contemporary physics is illustrated in the following passage from one of the most widely used textbooks on quantum field theory[1]. After showing that spacelike propagation is an inevitable consequence of the relativistic field equations, the authors say (p. 28):

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"To really discuss causality, however, we should ask not whether particles can propagate over spacelike intervals, but whether a *measurement* performed at one point can affect a measurement at another point whose separation is spacelike."

This passage clearly recognizes that, at the observational level, the standard projection and probability postulates constitute a completely causal and relativistic account. It also shows that the characterization of quantum field theory as incorporating the principle of causality at the most elementary level depends critically on the connection to macroscopic measurement. This critical dependence stems from the nonlocal effects that are inherent to quantum theory.

The reality of nonlocal projection effects was demonstrated by Bell[2, 3], and confirmed experimentally by Aspect[4, 5]. Bell showed that the correlations exhibited between the results of distant measurements on pairs of entangled particles could not be produced unless one of the measurements acts across spacelike intervals to affect¹ the other measurement.

The recognition of real nonlocal actions has led contemporary physics to modify the traditional concept of causality which implies strictly local propagation. The modern rendering of the principle as a prohibition of superluminal signaling reflects a significant generalization of the classical notion. This perspective allows nonlocal effects, provided that they cannot transmit information.

The differences between the classical and contemporary notions were discussed by Bell in his last essay[6]. His main arguments have been further elucidated by Maudlin[7] and Norsen[8]. Local causality captures our intuitive notion of physical processes propagating continuously through space, and is sufficient to insure that those processes can be described in a Lorentz covariant manner. Signal causality fails in both these respects. It is largely our inability to consistently describe projection as a process at all, or accomodate it readily within a relativistic spacetime that has made it difficult to construct a coherent microphysical explanation of measurement effects. This apparent obstacle to a fundamental explanation of measurement is what led Bell to express his distaste for the contemporary viewpoint.

Despite these legitimate concerns, signal causality, when construed as a relationship among purely physical processes and systems, is extremely useful as a guiding principle in the formulation of fundamental laws. Svetlichny[9, 10] has emphasized this point. He argues that superluminal communication is a generic property of physical theories. So those that exhibit strong nonlocal quantum correlations without signaling are very special. In Svetlichny's words, this property imposes a rigidity on fundamental theories. This rigidity means that the principle of causality is sufficient to determine many of the essential characteristics of the theories that embody it.

¹Bell's demonstration does not contradict the statement of Peskin and Schroeder since they are referring to effects on the *total* probability of an outcome, while Bell's correlations describe *conditional* probabilities of an outcome given specific results of distant measurements.

Note first that it is signal causality that has guided the development of quantum field theories and shaped some of their most fundamental properties. As a further illustration of this point consider the Born Probability Rule [11]. Given the nonlocal projection that results from measurement, the no-superluminal-signaling principle determines precisely how probabilities must be assigned to possible outcomes. (This is demonstrated in Gleason's theorem[12] since the no-superluminal-signaling requirement entails Gleason's crucial assumption of noncontextuality.) The rule is so familiar that we take it for granted, but as Bell points out [13] it is quite easy to imagine other reasonable prescriptions for ascribing probability. For example, one could make it proportional to the absolute value of the amplitude rather than the square of the amplitude. One could associate amplitudes with cosines and make the probability some function of the angle. Equal probabilities could be assigned to all nonzero amplitudes, or one could dictate a deterministic outcome based on the largest amplitude. The point is that any of these reasonable alternatives would enable transmission of information across spacelike intervals. The no-signaling principle selects the Born Rule as the *only* way to assign probabilities to outcomes.

Although the Born Rule is very specific, it does illustrate a very general, deep feature of causal, nonlocal theories, viz., indeterminism. The generality of the connection between nondeterminism and causality in the presence of nonlocal effects was brought into prominence over the last couple of decades through the efforts of a number of resarchers. Elitzur[14] argued that indeterminism should be taken as a fundamental principle of quantum theory that is deeply connected to both the prohibition of superluminal information transmission and the Second Law of Thermodynamics². Popescu and Rohrlich[17, 18, 19], building on earlier suggestions by Aharonov[20] and Shimony[21, 22], proposed taking nonlocality and relativistic causality as axioms, and deriving indeterminism as a consequence. Additional arguments that nonlocal effects must be nondeterministic with respect to observable quantities in order to prevent superluminal information transfer have been given in several works[9, 10, 23, 24, 25, 26, 27, 28]. The general relationship of causality, relativity, and observability to indeterminism has been eloquently summarized by Elitzur and Dolev[24]. They note that "Hidden variables must be forever-hidden variables". This view is reinforced by the work of Valentini [28] which shows that the prevention of superluminal signaling in the de Broglie-Bohm[29, 30] theory depends on the initial distribution of particles being consistent with the initial wave function. Other types of initial distributions predict experimental differences between Bohmian theory and standard quantum mechanics. The critical point is that these differences, if observed, would constitute superluminal signals.

Aharonov has described this situation as follows[31]:

"Instead of asserting, with Einstein, that 'God does not play dice!' we

²The possibility of a connection between measurement effects and entropy increase was also noted by von Neumann[15, 16]. The same work is also usually recognized as giving the first formulation of the Projection Postulate.

should ask ourselves, 'Why does God play dice?' i.e., What new possibilities does a non-deterministic universe offer? A first answer would be, non-determinism allows a universe that is self-consistent, and causal in the relativistic sense, to be nonlocal."

The picture that emerges is that the nonlocality of the Projection Postulate is tempered by the limits on determinism implied by the Born Rule to maintain causality.

By recognizing the limits on determinism as an essential feature of contemporary theory we solve one of the basic riddles of causality: How can real changes in definite physical states propagate across spacelike intervals without the transmission of information? The answer is that the limits on determinism imply that the concept of information is just not definable at the most elementary scales. To say that information about a physical state exists in a particular system implies that the state can be captured and reliably replicated. We know from the no-cloning theorem[32] that, in general, this cannot be done for elementary systems. It does not make much sense to say that an isolated elementary particle carries information if the laws of physics forbid the extraction of that information³. The underlying reason that one cannot find out everything about arbitrary states is that the physical processes that constitute measurements are partially nondeterministic. The concept of information requires a degree of determinism that, in general, just does not exist in interactions involving a few elementary particles. It takes a large number of such interactions to insure that an outcome of a probabilistic process is fully resolved. The relevant properties must also be reflected in the states of enough individual particles so that the record of the outcome is not wiped out by further probabilistic occurrences. Information can only be said to meaningfully exist at a scale at which statistical stability is reasonably assured.⁴

So the no-superliminal-signaling principle entails the Born Rule (given the vector space structure of quantum theory and nonlocal projection effects). This precludes violations of macroscopic causality, and the indeterminism that is implied also explains how real physical changes can propagate across spacelike intervals without the transmission of information. The indeterminism prevents information from being instantiated in isolated elementary systems.

But there is at least one further implication of signal causality that provides a critical clue in constructing a microphysical explanation for nonlocal measurement effects. As already emphasized, to serve as a useful guide, this principle must be interpreted as a relationship among *physical* entities without any reference to intelligent observers. Therefore, any physical process that can observe the results of nonlocal

³Verification measurements that register previously prepared states do not serve as counterexamples to this point. Particles in prepared states are not isolated; information is instantiated in their relationship to the preparation apparatus and process.

⁴The situation is somewhat analogous to the relationship between statistical mechanics and thermodynamics where the concepts of temperature, pressure, and entropy are defined only for systems that are large enough to approximate thermal equilibrium.

effects such as wave function collapse must, itself, be capable of inducing those effects. The physical processes that bring about collapse are measurements. If measurements did not themselves bring about the nonlocal effects, they would be observing the results of *other* physical processes that occur at spacelike separation. Aside from any question about whether intelligent observers are signaling one another, information about the occurrence of those other processes would be transmitted across spacelike intervals, violating causality. To avoid such violations, one must assume that it is something *intrinsic to the measurement process* that induces the nonlocal quantum effects that are observed.

The connection between measurement and projection, is, of course, explicitly spelled out in the Projection Postulate, but the fact that it is a critical implication of causality is often overlooked. To maintain a meaningful notion of causality at the microphysical level we should assume that nonlocal collapse effects result from the interactions that establish correlations between elementary particles. These are what measurements are made of. When these interactions involve part, but not all, of the of the subject wave functions, they can generate entanglement and the problematic, nonlocal effects associated with it. In the hypotheses offered here, these *elementary entangling interactions* will be responsible both for creating entanglement relations and for the projection-like effects that eventually break them.

For the proposed explanation to make sense, all that we need to assume is that entanglement, the generation of entanglement, and the nonlocal correlations implied by it are genuine physical phenomena. The reality of these effects has been well established, both theoretically and experimentally, and they constitute the truly distinctive characteristics of quantum theory. This approach allows us to get to the heart of the problem without getting hung up on specific aspects of the formalism that might eventually be superseded. We will also see that this assumption solves the so-called "basis selection problem" in a very simple and natural way.

The account of nonlocal collapse effects that I will offer turns on a willingness to acknowledge (at least provisionally) that current theory assumes that there are essential limits to determinism, but these limits do not call into question the reality of elementary processes. The need to distinguish between physical realism and determinism is eloquently described by Bradley[33]. After expressing a generally positive view of determinism he says:

"Nevertheless, I have to admit that there is no a priori reason why either a metaphysical or scientific realist should be a determinist. Neither form of realism actually entails determinism. Perhaps God does play dice with the cosmos after all. That the universe should be indeterministic - that events at the microphysical level, in particular, should be uncaused - is entirely conceivable."

Gisin[34, 35] has also discussed the need to keep the distinction clear, and pointed out some of the confusions that arise when these two different concepts are conflated.

For physicists, it is reasonable to take realism (a belief in the objective existence of external reality) as an axiom, but determinism should be treated as an hypothesis. Hume[36] made this point over 250 years ago when he pointed out that there are no necessary connections in experience.

To summarize, given the real nonlocal effects pointed out by Bell's analysis, the principle of signal causality entails that the processes that observe these effects must be the ones that induce them, and that they must be nondeterministic with probabilities in accord with the Born Rule. From this perspective, the standard measurement postulates are straightforward consequences of the prohibition on superluminal information transmission. When we translate these implications to a more fundamental level we see that the nondeterministic aspects of elementary interactions prevent information from being instantiated in very small physical systems, and that the nonlocal effects must be induced by elementary entangling interactions.

These inferences will be exploited in Section 3 to develop a microphysical account of nonlocal measurement effects. But first we must try to reconcile our understanding of relativity and spacetime structure with such nonlocal effects.

2 Causality, Relativity, and the Sequencing of Nonlocal Effects

Causality and relativity are intimately related. In elementary discussions, this close relationship is usually attributed to spacetime structure, and it is easy to see how both properties are maintained when physical processes are confined to the light cone. Above, we saw that, with nonlocal measurement effects, the preservation of causality entails the Born Rule. Given the close connection between the two principles, we should expect that relativity also depends on the rule. This dependence can be illustrated with a simple two-particle entangled system.

Consider a situation with identical particles in the state: $\alpha |x_1\rangle |x_2\rangle + \beta |y_1\rangle |y_2\rangle$, with $|x\rangle$ and $|y\rangle$ orthogonal, and $\alpha\alpha^* + \beta\beta^* = 1$. Define an alternate basis for the second particle:

$$|u_2\rangle = \gamma |x_2\rangle + \delta |y_2\rangle, |v_2\rangle = \delta^* |x_2\rangle - \gamma^* |y_2\rangle.$$

The original basis can be expressed as:

$$|x_2\rangle = \gamma^*|u_2\rangle + \delta|v_2\rangle, |y_2\rangle = \delta^*|u_2\rangle - \gamma|v_2\rangle.$$

One can represent the first particle in the $|x\rangle, |y\rangle$ basis, and the second particle in the $|u\rangle, |v\rangle$ basis:

$$\alpha |x_1\rangle (\gamma^* |u_2\rangle + \delta |v_2\rangle) + \beta |y_1\rangle (\delta^* |u_2\rangle - \gamma |v_2\rangle)$$

$$= \alpha \gamma^* |x_1\rangle |u_2\rangle + \alpha \delta |x_1\rangle |v_2\rangle + \beta \delta^* |y_1\rangle |u_2\rangle - \beta \gamma |y_1\rangle |v_2\rangle.$$

In Bell-EPR[2, 3, 37]-type experiments one can measure either particle in any basis whatsoever. One can also measure one of the particles without measuring the other.

The no-superluminal-signaling principle implies that a measurement on one particle must not affect the outcome probabilities of measurements on the other. Therefore, the sum of probabilities conditioned on specific outcomes of a measurement must equal the total probability when no measurement is made on the other particle. Specifically, for a $|u\rangle$ outcome on the second particle they must satisfy:

$$P(\alpha)P(\gamma^*) + P(\beta)P(\delta^*) = P(\alpha\gamma^*) + P(\beta\delta^*),$$

where $P(\alpha)$ denotes the probability associated with the amplitude, α . This condition is obviously fulfilled by the Born Rule $(\alpha\alpha^*\gamma\gamma^* + \beta\beta^*\delta\delta^*)$, and it is easy to show that this is the *only* way to satisfy it.⁵ This equality has been derived from the assumption of *causality*, i.e., that the total outcome probability of a measurement on particle 2 is not affected by a measurement on particle 1. However, one can also take it as a statement that the sequence of measurements can be freely interchanged, since the right-side expression implies that the measurement on the second particle is made first, and the left side can be read in either order. This freedom to interchange the sequence of spacelike-separated events is a hallmark of *relativity*. So, relativity, like causality, is *not* simply a consequence of spacetime structure. In describing nonlocal measurement effects, it is dependent on the specific form of indeterminism embodied in the Born Rule. A very similar point is made by Elitzur and Dolev[24].

Relativity is a property of our *theories*. It is rooted in several deep features of those theories - not just space-time relationships. These features include fundamental limits on observability that are tied to nondeterministic effects of elementary interactions.

We have seen that causality requires that nonlocal measurement effects must be probabilistic. This nondeteminism, in turn, implies that the effects are irreversible. It is extremely difficult to construct a coherent account of such irreversible effects without assuming that they occur in some definite sequence. I will shortly offer additional arguments that we should attribute some objective, though unobservable, sequence to these nonlocal effects.

Such objective sequences of spacelike-separated events are, of course, at odds with our ideas about relativity. This apparent conflict can be resolved in essentially the same way in which one reconciles the propagation of real nonlocal effects with the impossibility of superluminal information transmission. The effects of the sequencing of nonlocal actions are not observable because those actions occur at a level well below that at which the relevant information could be instantiated. Observations are macroscopic, or at least, mesoscopic processes that involve the recording of reasonably definite outcomes of a series of probabilistic events. Any single observation is consistent with a wide range of sequences of nondeterministic microphysical events. It is the many-to-one map of possible sequences to a particular observed outcome that makes

⁵See Gleason's theorem[12].

 $^{^6}$ The connection between indeterminism and irreversibility has been pointed out in several places by Elitzur[14], and by Elitzur and Dolev[24, 25].

⁷Maudlin[7] has pointed out many of the complications for such attempts.

a relativistic description appropriate for situations with multiple spacelike-separated measurements.

So, in measurement situations, relativistic descriptions do not capture all spacetime relationships because some of those relationships leave no physical trace. Our ability to apply such descriptions depends, in large part, on in-principle limitations to the amount of information generated by elementary processes.

The idea that relativistic symmetries reflect limits on information is not necessarily tied to a fundamental indeterminism. A proposal by 't Hooft[38] is based on the idea that there is an essential information loss mechanism between the most basic ontological level and the level of quantum description:

"We ... argue that symmetries such as rotation ..., translation, Lorentz invariance, ... gauge symmetries and coordinate reparametrization invariance, might all be emergent... [and] that information is not conserved in the deterministic description."

While deterministic extensions of probabilistic theories are always possible, in principle,⁸ the goal here is to outline a logically coherent microphysical explanation of measurement effects within the framework of contemporary physics. Causality and relativity are defining features of that framework. Their preservation requires that we accept the Born limit on determinism as a guiding principle. This perspective allows us to view causality and relativity as consequences of fundamental physical law, even though they do not derive only from the structure of spacetime.

The need to invoke the limits on determinism and information in order to save relativity stems from the fact that there is an implied sequencing of the nonlocal effects described by the measurement postulates. Although nonlocal propagation, in itself, can present challenges to the construction of a relativistic description of physical processes based solely on the structure of spacetime, these challenges are not necessarily insurmountable. Some aspects of nonlocality can be dealt with, as indicated in the passage from Peskin and Schroeder[1] quoted earlier, and as discussed at some length by Maudlin[7]. However, attribution of objective sequences to large sets of spacelike-separated events creates extreme difficulties for the notion that relativity concerns only spacetime relationships. Let us now look at some of the reasons that one would want to posit such sequences, even though they force us to alter some of our ideas about the basis for relativity.

I will first look at the issue of sequencing as it applies to macroscopic measurements, and then extend the analysis to the elementary interactions that constitute those measurements. Recall the simple entangled state described above. The state

⁸ One can always "fill out" the predictions of a probabilistic theory by providing a (possibly infinite) list that determines everything that was initially left undetermined. Interesting deterministic extensions generate the list dynamically from initial conditions.

⁹The need to assume some sort of sequencing in any logically coherent account of measurement effects has been noted frequently in the literature[39, 7]

can be represented in two ways depending on which measurement one wants to emphasize.

$$\begin{array}{lcl} \alpha|x_1\rangle \,\otimes\, (\gamma^*|u_2\rangle + \delta|v_2\rangle) & + & \beta|y_1\rangle \,\otimes\, (\delta^*|u_2\rangle - \gamma|v_2\rangle) \\ = & (\kappa|x_1\rangle| + \lambda|y_1\rangle) \,\otimes\, \sigma|u_2\rangle & + & (\mu|x_1\rangle| + \nu|y_1\rangle) \,\otimes\, \tau|v_2\rangle. \end{array}$$

The relevant relationships between the two sets of coefficients are: $\alpha \gamma^* = \kappa \sigma$, $\beta \delta^* = \lambda \sigma$, $\alpha \delta = \mu \tau$, and $\beta \gamma = -\nu \tau$.

Suppose that measurements on the two particles are *timelike* separated. A measurement on particle 1 yields an $|x_1\rangle$ result, and a later one on particle 2 produces a $|u_2\rangle$ outcome. The sequence of projections is:

$$\begin{array}{ccc} \alpha|x_1\rangle \otimes (\gamma^*|u_2\rangle + \delta|v_2\rangle) & + & \beta|y_1\rangle \otimes (\delta^*|u_2\rangle - \gamma|v_2\rangle) \\ \Longrightarrow & |x_1\rangle \otimes (\gamma^*|u_2\rangle + \delta|v_2\rangle) & \Longrightarrow & |x_1\rangle|u_2\rangle. \end{array}$$

If the order of the measurements were reversed the projection sequence would be:

$$(\kappa|x_1\rangle + \lambda|y_1\rangle) \otimes \sigma|u_2\rangle + (\mu|x_1\rangle + \nu|y_1\rangle) \otimes \tau|v_2\rangle$$

$$\Longrightarrow (\kappa|x_1\rangle + \lambda|y_1\rangle) \otimes |u_2\rangle \Longrightarrow |x_1\rangle|u_2\rangle.$$

The correlations between the outcomes are encoded in the joint probabilities. The sequential projections provide a very simple way to understand these correlations. Because $P(\alpha) * P(\gamma^*) = P(\kappa) * P(\sigma)$ they are not affected by reversing the order of the measurements.

In these timelike-separated cases we can apply the Projection Postulate in a completely straightforward manner. The first measurement collapses the wave function to the observed outcome and to the state of the other particle that is correlated with that outcome. The second measurement then acts on the state to which the system has collapsed. The measurements act (possibly nonlocally) on the state by interacting locally with one of the branches of the wave function. Nonlocal effects are mediated by entanglement relations. The nonlocal effects are at odds with our classical intuitions about causality, but the logical relationships are quite simple.

We can retain this straightforward picture for situations in which measurements are spacelike-separated if we are willing to attribute *some* sequence to those measurements. The Bell-EPR correlations between outcomes are the same as those just described for the timelike-separated measurements. As just shown, the joint probabilities of outcomes (and, hence, the correlations) are not affected by which measurement is sequenced first. The Born Rule prevents any observable superluminal effect of one measurement on the other, and it also precludes any detection of what the sequence is. Note that, with the assumption of sequencing, the only nonlocal action is by the measurement on the state. The measurements do not need to cooperate, conspire, or communicate with each other across a spacelike interval in order to maintain the Bell correlations.

This last point is relevant because the goal here is to outline a logically simple microphysical explanation of wave function collapse, viewed as a real physical process.

That explanation will consist of two assumptions. One of these concerns the sequencing of spacelike-separated elementary, entangling interactions; the other concerns the nondeterministic, nonlocal action of those interactions on the wave functions of the interacting particles. One might think that, in the interest of preserving more of the relativistic spacetime structure, it would be desirable to dispense with the sequencing assumption, and posit some additional type of nonlocal effect that enables spacelike-separated measurement processes to cooperate. However, it is very difficult to develop any explanation that does not make some analogous assumption about the ordering of events in spacetime that is seriously at odds with conventional ideas about relativity.

In situations with just two well-defined, localized, spacelike-separated measurements, it is tempting to view them as acting jointly in the reference frame in which they are simultaneous, to produce a single projection to the final state:

$$\alpha \gamma^* |x_1\rangle |u_2\rangle + \alpha \delta |x_1\rangle |v_2\rangle + \beta \delta^* |y_1\rangle |u_2\rangle - \beta \gamma |y_1\rangle |v_2\rangle \quad \Longrightarrow \quad |x_1\rangle |u_2\rangle.$$

However, if more than two processes are involved, such a frame does not always exist. Formally, one can still view the sets of spacelike-separated interactions as constituting a single measurement process. ¹⁰ But the problem becomes more complicated when we consider mixed sets of interactions, involving both spacelike and timelike separations between them. Although they can still be viewed, from a formal point of view, as one measurement process, this would take us even farther from an understanding of the nonlocal actions as real, physical effects.

To see this, suppose that a set, A, of interactions is timelike earlier than B, and that both A and B are spacelike-separated from C. Considered by itself, we would expect A to yield a definite outcome, as a result of some physical process. To suggest that this outcome must be postponed until the ocurrence of B (and possibly even later events) because of something that might be happening at a spacelike separation is counterintuitive, and makes any potential physical explanation much more convoluted. It also raises the possibility of indefinite further postponements. (A similar point about indefinite regress is made by Maudlin[7] concerning Cramer's transactional interpretation[40].) To try to associate timelike-separated processes with distinct projections assumes some criteria for grouping processes across spacelike intervals: should C be bracketed with A or with B? Such grouping criteria would conflict with relativistic structure just as much as the assumption of sequencing of spacelike-separated events.

The motivation to assume some type of sequencing becomes even stronger when one contemplates possible explanations of projection in terms of elementary interactions. Any real, measurement process consists of myriad elementary correlating interactions, with both spacelike and timelike relations to one another. So, even in the case of a single, reasonably well-defined macroscopic measurement, we would face

¹⁰Thanks to the reviewer for emphasizing this point.

all of the issues described in the previous paragraph. One should also note that entanglement relations are generated by exactly these kinds of interactions, and that these entanglement relations both mediate and define wave function collapse.

This last point implies that the generation of entanglement, itself, is an important type of nonlocal action. Although the entangling effects are deterministic, they define the scope of the collapse effects which are not. At a fundamental level, the entanglement relations determine exactly which elementary systems are directly affected by the probabilistic projections. The fact that causality entails that the projection effects are tied to the entangling interactions helps to simplify the overall picture. But it also means that even simultaneous interactions involving the same entangled system must be sequenced.

To make clear just what is being proposed, let us compare the hypothesized sequencing to the de Broglie - Bohm theory[29, 30]. Bohmian mechanics assumes an absloute time (at least in the most straightforward version). This is obviously at odds with the idea of a relativistic spacetime, and it implies much stronger temporal ordering relations. However, it still allows large numbers of entangling interactions to occur simultaneously. The proposal outlined in the next section associates these individual elementary interactions with the collapse effects. Because these effects are probabilistic, and, hence, irreversible, logical simplicity strongly suggests that even simultaneous interactions should be assigned some objective (though indiscernible) sequence. The reason that Bohmian theory is able to present a logically coherent account of measurement outcomes is that, in it, the wave function never collapses. The outcomes are determined by the action of the wave function in conjunction with the simultaneous positions of all of the particles involved. Portions of the wave function become irrelevant¹¹, but they never vanish.

The sequencing of spacelike-separated entangling interactions that is proposed here goes beyond what is implied by a notion of absolute time. For the reasons described earlier, we want to insure that entanglement relations are well defined at the time and place of each individual interaction, and we also want to avoid the need for interactions to cooperate across spacelike intervals. Therefore we will assume that, no matter what spacelike hypersurface is given, there is some objective (though unobservable) sequence of the nonlocal effects of interactions on that surface that involve the same entangled system. So if A and B are spacelike-separated interactions involving particles that are entangled with one another, either A is "prior" to B, or B is "prior" to A.

There are some similarities between this proposal and Bohmian theory. One can describe the sequencing by reference to a set of preferred (unobservable) spacelike hypersurfaces. These need not be hyperplanes, as they are in the de Broglie - Bohm account. However, a more critical difference is that the set of hypersurfaces does not form a foliation of spacetime, because successive surfaces will not be completely

¹¹Because they are so far removed from the system location in configuration space.

disjoint. To describe the idea more precisely, let us consider two spacelike-separated interactions, A and B, that involve the same entangled system. Imagine that the evolving hypersurface lies immediately earlier than both A and B. If the nonlocal effects of interaction A are to be sequenced prior to those of B, then the surface should be pushed forward in the local vicinity of A, but held back in the region of B. This situation can be characterized with a parameter, s. Let s_0 label the hypersurface that was just described, and let s_1 label the surface that is pushed forward to include A, but remains stationary in the neighborhood of B (and, hence, is still earlier than B). The surface that is eventually pushed forward to include B can be labeled s_2 . The sequencing of A prior to B is reflected in the fact that $s_1 < s_2$ (since s_1 labels the first surface on which A lies, and s_2 labels the first surface to include B). s_2

This approach is closely akin to the many-time formalism of quantum theory[41, 42]. This formalism has been exploited by Bell[43], by Bohm and Hiley[44], and by Bedingham[45] to deal with the issues discussed here. However, it is important to (again) note that we do not revert to a foliation of spacetime for simplicity, as is sometimes done. The idea is to exploit the full freedom allowed by the multiple-time approach in order to allow for a more complete sequencing of the relevant interactions.

This formulation enables one to describe the nonlocal effects of each interaction involving the same entangled system individually, in sequence. This allows us to construct a logically simple account of wave function collapse. On any particular surface, entanglement relations and states are well defined. The nonlocal effects are induced by the interaction on the state, and they propagate along the surface. The specific nature of the effects (to be described in the next section) reproduces the standard quantum probabilities and correlations, and it insures that sequencing of spacelike-separated interactions is unobservable. With respect to a standard foliation of spacetime, the family of surfaces hypothesized here is overcomplete. Any particular event can lie on many such surfaces.

This description of nonlocal collapse effects is *not* Lorentz-invariant or covariant. The idea is that a standard relativistic account can be recovered when one averages over all of the possible families of evolving surfaces (and sequences) that are consistent with the observed outcome. This averaging is roughly similar to a statistical mechanics description that averages over many possible microconfigurations.

The evolving surface described here is intended to be relevant only for the description of nonlocal collapse effects. Ordinary deterministic processes are constrained to propagate within the light cone and are not affected by this additional spacetime structure.

The evolution of the surface is constrained by entanglement relations in order to insure that interactions affecting a given system are sequenced one at a time, and by the requirement that it must remain spacelike. Other than this, the evolution is assumed to be random. So, just as in conventional formulations of relativity, there is

¹²We can fill out the description by using local time variables $d\tau_A$ and $d\tau_B$. Since A is sequenced first, we get $d\tau_A/ds > 0$, $d\tau_B/ds = 0$ during the interval $s_0 < s < s_1$.

no preferred reference frame.

I will not attempt to completely fill out this picture here. Instead, I will consider discrete sets of interactions. An hypothesis concerning the nonlocal effects of such interactions is presented in the next section. It will reproduce the Projection Postulate and the Born Probability Rule at the macroscopic level, and it will entail (due to the indeterminism of the nonlocal effects), that every possible sequence of spacelike-separated interactions is consistent with the observed outcome. The assumption that there is an objective sequence of these interactions provides logical coherence to the account. The fact that the sequences are unobservable maintains consistency with relativity.

3 Connecting Measurement to Elementary Interactions

Measurements are essentially detections, or failures to detect. Ultimately, they register the presence or absence of particles. The state vector collapse that is observed implies the transfer of the wave function's amplitude either completely into or out of the region of the interactions that constitute the measurement. Cases in which the correlating interactions involve the complete wave function of the measured system correspond to simple verification measurements, i.e., those in which the system is already in an eigenstate of the measured observable. It is when some of the relevant interactions involve part, but not all, of the subject wave function, that the problematic, nonlocal transfers of amplitude can occur. ¹³

It is the interactions that also *define* the measurement basis. At a macroscopic level the interaction Hamiltonian of the measurement apparatus determines what is being measured. This has been clearly shown by Laura and Vanni[46]. They demonstrate that different Hamiltonians measure different observables.¹⁴

At an elementary level, the interactions determine the basis simply by defining a bifurcation of the wave function into interacting and noninteracting parts. In this respect the individual interactions exhibit a clear parallel to the "yes-no" detection-like character of macroscopic measurements. This close parallel suggests a very straightforward hypothesis about how elementary entangling interactions eventually bring about wave function collapse. Since collapse is equivalent to the complete transfer of amplitude either into or out of the measurement region, it is natural to suppose that each individual interaction transfers some small amount of amplitude either into or out of the state resulting from that interaction. With enough such shifts, even-

¹³These "partial" encounters are one of the principal types of interactions that generate entanglement. Processes such as parametric down conversion in which particles are *created* in an already entangled state are the other principal type.

¹⁴These arguments are related to earlier ones developed by Zurek[47], in his description of the decoherence approach.

tually all of the amplitude will be transferred. (Since different interactions can split the wave function in different ways, macroscopic measurements can have arbitrarily many possible outcomes.) Note that it is the *linearity* of the Schrödinger equation that permits nonlocal shifts in amplitude without otherwise disturbing the ordinary deterministic evolution of the system.

Of course, the shifts must reproduce the Born Rule at the level of macroscopic outcomes, but this is not difficult to arrange. According to the rule, the probability of an outcome is equal to the absolute square of the amplitude of the relevant wave function component, so it is convenient to give this quantity a name¹⁵. Since these quantities occur as the diagonal elements of density matrices, we can refer to them as "Born densities", or, simply, as densities.

With this terminology, and given the motivations described above, wave function collapse can be connected to elementary entangling interactions between pre-existing particles by the following incremental density shift hypothesis. Consider the binary splitting of the wave function of the combined system into interacting and noninteracting components. Label the Born density of the interacting component as p; label the density of the complementary component as q=1-p. Associated with each individual entangling interaction of an elementary particle, there is a small nonlocal shift, d, in the quantities p and q, such that $p \to p + d$ and $q \to q - d$, OR $p \to p - d$ and $q \to q + d$. The shift is unbiased; that is, the probability of an increase in p (and corresponding decrease in q) is equal to the probability of a decrease in p (and corresponding increase in q). The linearity of the ordinary deterministic equations permits such amplitude redistribution without disturbing relative phases or producing other changes which could enable superluminal signaling.

The size of the density shifts, d, is assumed to be some small number. It cannot exceed either p or q, since p and q are positive numbers bounded by 0 and 1. Because this is a phenomenological account, constructed to reproduce the macroscopic postulates, there are few other constraints that can be placed on it here. In particular, it might vary from one interaction to the next. An estimate of the typical size of the shifts will be given in the next section based on indirect arguments.

The hypothetical small, nonlocal, nondeterministic effects are in addition to the standard effects of such interactions, which are described within conventional quantum mechanics (or quantum field theory). The overall collapse process consists of numerous small random density shifts between branches of the entangled system. Since an entangled system increases in size as the interactions proceed, the scope of the density shifts increases to include more and more particles in the collapse process. Eventually the scope can approach mesoscopic scales. The expansion of the entangled system also allows many additional particles to participate in the interactions (and

¹⁵Calling it a "probability" would beg the question, and cause confusion, since we want to *prove* that it equals the probability.

¹⁶This hypothesis is somewhat similar to the account proposed recently by Bedingham[45]. It is also essentially identical to one made in an earlier unpublished work by this author[48].

density shifts). The particular way in which the system is split can change from one interaction to the next, since different interactions can define different splittings of the wave function. This makes possible measurement processes with arbitrarily many possible outcomes. As a result of the random density shifts or of the experimental arrangement, one particular decomposition eventually becomes dominant, and all amplitude is shifted to one of the branches in that pair. For any specific binary decomposition, a sequence of steps toward one or the other component has the character of a random walk (with possibly varying step size). If \overline{d} is the average step size, then a typical walk will terminate in about $(1/\overline{d})^2$ steps. So the hypothesis reproduces the Projection Postulate in a very straightforward way.

Since the density shift proposal was constructed specifically to reproduce the Born Rule, showing that it does so is fairly simple. There are three types of cases to consider. The simplest situations are those in which the wave function is divided into two principal branches, and the particle is detected in one of them. The decomposition defined by the interactions remains the same throughout the collapse process. ¹⁷ More general measurement situations allow for different ways of splitting the wave function at various stages of the process. The second category covers experiments with more than two chains of detector particles, and hence more than two possible mutually orthogonal outcomes. The third type includes Bell-EPR[2, 3, 37] experiments (and their generalizations) in which different (measurement) bases can be chosen in spacelike-separated regions. The distinctive feature of this third type of case is that the states resulting from the measurements can be distinct without being completely orthogonal.

The simplest types of measurements consist of sequences of elementary entangling interactions sufficiently long to determine which of two orthogonal possibilities is realized. Detectors can be placed in either one or both of the branches. The key point for these cases is that the binary decomposition of the system defined by the interactions of either of the detectors remains the same throughout. Label the two branches as A and B. For the given decomposition, let p_0 designate the initial value of the density of branch A. According to the Born Rule the probability that a sequence of correlating interactions long enough to constitute a measurement will produce this component as the outcome is just p_0 . We can describe the collapse brought about by the measurement as a change in the density from p_0 to 1.0, or from p_0 to 0.

Let $Pr(p_0)$ designate the probability of an A outcome predicted by the shift hypothesis in this kind of arrangement. We want to show that $Pr(p_0) = p_0$. Note first that the boundary conditions are determined by the requirement that the step size, d, cannot exceed the smaller of p and q. Since the density lies between 0 and 1, it is obvious that Pr(0) = 0, and Pr(1.0) = 1.0. Given that there is a 0.5 probability of a

¹⁷By "remains the same" I am referring to the decomposition of the initial entangled system; as entangling interactions occur the number of particles correlated with the initial branches of the wave function increases.

step in either direction, and that the step size is the same in either direction we get:

$$Pr(p_0) = \left(\frac{1}{2}\right) Pr(p_0 + d) + \left(\frac{1}{2}\right) Pr(p_0 - d)$$
 (3.1)

(for any $d: 0 \le d \le p_0, 0 \le d \le q_0$). This expression represents an infinite set of difference equations. One solution is $Pr(p_0) = p_0[49]$. This clearly satisfies both the boundary conditions and Eqn. 3.1.¹⁸ It is straightforward to show that the solution is unique. So, for a sequence of elementary interactions in which the binary decomposition defined by those interactions does not change, the shift hypothesis yields the Born Probability Rule.

The second category includes standard many-outcome (i.e., more than two) experiments. The fact that the possible outcomes are all mutually orthogonal keeps the analysis simple. For purposes of illustration we can imagine that there are several well-defined branches of the wave function, and that there is a detector in each branch. With this kind of set-up, at each elementary stage, an interaction from one particular detector is selected (as described in Section 2). The branch corresponding to this detector will constitute one of the principal components of the binary decomposition defined at this stage; we can label this component as O_i . The complementary component can be designated O'; it consists of all branches O_i with $i \neq i$. Let us look at one of these other branches, O_k . Assume that the density of $O_i = p$, that of $O'=q \ (=1-p)$, and that of $O_k=r$. The incremental shifts associated with the elementary interactions will change the densities of O_i and O' to either p+d and q-d, or p-d and q+d. Since O_k is completely contained in O', we know that $r \leq q$. The transfer of density between the two orthogonal components of the wave function defined by the interaction must leave all phase relationships within each principal component undisturbed. This implies that all relative densities within a main branch will remain the same for that interaction. Hence the change in the overall density of a major component is distributed proportionately among its sub-branches. Therefore, the shift in r will be (r/q)d = d'. Since d is the same whether q is increased or decreased, d' must also be the same whether it constitutes an increase or a decrease. So from the point of view of the O_k component, this shift can be viewed as just part of its own random walk with a different step size. Since we have explicitly allowed for varying step sizes in the course of the walk, the probability that a complete measurement sequence will result in O_k is the same as that demonstrated above. So the Born Rule also holds for these kinds of arrangements.

The third category involves Bell-EPR type experiments and their generalizations. In these situations the binary decompositions of the wave function defined by spacelike-separated interactions result in branches that overlap without being identical. This means that the possible outcomes of the measurements might be neither identical nor orthogonal. To establish the Born Rule for these cases one must show

¹⁸An essentially equivalent discussion is given in [48]; a similar one is given in [50].

that steps in a random walk in one binary decomposition do not bias possible outcomes in a different, overlapping decomposition. I will do this by showing that no matter what decomposition is considered, the densities of the components in that basis undergo the same kind of unbiased random walk as that described above.

Typical Bell-EPR experiments involve two-particle systems with two states each. If $\alpha = -\beta = 1/\sqrt{2}$, and if $|x\rangle$ and $|y\rangle$ represent up and down spin states, then a two-particle singlet state can be represented as:

$$\alpha |x\rangle |y\rangle + \beta |y\rangle |x\rangle.$$

With an alternate measurement basis for the second particle,

$$|u\rangle = \gamma |x\rangle + \delta |y\rangle, \quad |v\rangle = \delta^* |x\rangle - \gamma^* |y\rangle,$$

we get:

$$\begin{aligned} &\alpha|x\rangle|y\rangle + \beta|y\rangle|x\rangle \\ &= &\alpha|x\rangle\otimes(\delta^*|u\rangle - \gamma|v\rangle) + \beta|y\rangle\otimes(\gamma^*|u\rangle + \delta|v\rangle) \\ &= &\alpha\delta^*|x\rangle|u\rangle - \alpha\gamma|x\rangle|v\rangle + \beta\gamma^*|y\rangle|u\rangle + \beta\delta|y\rangle|v\rangle. \end{aligned}$$

Suppose that a measurement interaction involving the first particle results in a step of size d toward the $|x\rangle$ state. The density of this state increases from $\alpha\alpha^*$ to $\alpha\alpha^*+d$; the density of the $|y\rangle$ state for the first particle decreases from $\beta\beta^*$ to $\beta\beta^*-d$. As a result of this shift, the density of the $|u\rangle$ state for the second particle changes from $\alpha\alpha^*\delta\delta^*+\beta\beta^*\gamma\gamma^*$ to $(\alpha\alpha^*+d)\delta\delta^*+(\beta\beta^*-d)\gamma\gamma^*$. The net change for $|u\rangle$ is $d'=d(\delta\delta^*-\gamma\gamma^*)$. The density of the $|v\rangle$ state changes from $\alpha\alpha^*\gamma\gamma^*+\beta\beta^*\delta\delta^*$ to $(\alpha\alpha^*+d)\gamma\gamma^*+(\beta\beta^*-d)\delta\delta^*$, a net change of $d(\gamma\gamma^*-\delta\delta^*)=-d'$.

So the densities for the orthogonal $|u\rangle|v\rangle$ states change by the same amount in opposite directions. If the interaction of the first particle had produced a step of size d toward the $|y\rangle$ state, we would have gotten an induced step in the $|u\rangle|v\rangle$ basis of identical size but with the opposite sign. Again, we can view this as a step in an unbiased random walk in the $|u\rangle|v\rangle$ basis, so the derivation of the Born Rule given above still applies. It is possible to define more general entangled states. Extension of the proof to these situations is straightforward.

By looking at each elementary entangling interaction as defining a two-way splitting of the system, and allowing for equally probable shifts of density between the two principal components, we have seen that we can view any component in any basis whatsoever as participating in a random walk in which its density, p, moves between 0 and 1. Its probability of reaching 1.0 is just p (if the "walk" is sufficiently long). Thus, the density shift hypothesis entails the Born Rule in all simple measurements and in all cases in which multiple, commuting measurements take place, even when different bases (corresponding to different orthogonal decompositions) are selected by detectors in spacelike-separated regions. So the density shift hypothesis reproduces the predictions of standard quantum theory in all situations in which questions of causality arise.

Conservation laws are maintained in the same way as in conventional theory. They are enforced exactly and locally by the dynamic equation governing Schrödinger evolution. When measurementlike interactions take place, the nonlocal density shifts conserve the relevant quantities in a probabilistic sense, i.e., the expectation value of a change in the quantity is zero.¹⁹

The discussion has centered on "measurements", but this does not mean that collapse only occurs in laboratories when measurements are made. Under the density shift hypothesis, collapse does not require a specially designed detection apparatus, or the intervention of an intelligent observer. Elementary entangling interactions take place all the time, everywhere. Whenever the wave function of a particle splits into several branches, or becomes sufficiently spread out, so that an interaction with another elementary particle does not engage the entire wave function, entanglement results, and density shifts occur. A sufficient number of such shifts produce complete state vector reduction.²⁰

In summary, it is possible to construct a microphysical explanation of nonlocal measurement effects by attributing those effects to elementary correlating interactions. By assuming that the processes that induce wave function collapse are the same ones that observe it, we help to preserve causality. By focusing on the interactions that constitute measurements we see that they both generate entanglement and eventually break it. They also define a natural bifurcation of the wave function that explains how the measurement basis is selected; this bifurcation suggests a simple density shift hypothesis. If we are willing to acknowledge that the density shifts that result in state vector reduction are essentially nondeterministic, then we have a complete explanation of how causality follows from the fundamental laws of physics, and why information can only be realized at mesoscopic and larger scales. The inprinciple unobservability of sequences of spacelike-separated interactions also follows as an immediate consequence. It is obvious that given any measurement result and any sequence of shifts, we can assign directions to the shifts that yield the observed result. Since observations are inherently mesoscale or larger processes, they must be consistent with any sequence of spacelike-separated elementary interactions. This is how relativity is preserved.

¹⁹In situations such as certain types of scattering experiments, additional constraints can insure exact conservation of some quantities.

²⁰The view that the registration of measurement results is an *objective* process, involving ordinary physical interactions, and not dependent on the presence of an intelligent observer, has been expressed by many. Notable among these is Bohr, who mentions irreversible acts of amplification[51]. But it is important to emphasize that the view that I am advocating is that the elementary interactions are also perfectly objective, though not deterministic, events. This latter view is not shared by all.

4 Estimating the Collapse Scale

In order to design experiments and assess their feasibility we need some idea of the size of the deviations from linear evolution hypothesized in the previous section. Because this is a phenomenological account we cannot derive this from first principles, but we can make an indirect estimate. The density shift hypothesis states that the size, d, is less than or equal to the density of the interacting component, p, and the complementary density, q = 1 - p. Quantum eraser experiments and quantum computer implementations involving a few elementary systems have shown that superposition effects persist after small numbers of entangling interactions[52, 53, 54, 55]. So any deviations from perfect linearity must be fairly small compared to the typical amplitudes of the particles involved in these experiments. (This point will be further elaborated in Section 5.)

Estimates of the collapse scale have been given in previous works that treat non-local projection effects as real physical phenomena. The dynamical reduction theory of Ghirardi, Rimini, and Weber (GRW)[56, 57], and the works of Pearle[50, 58] suggest tentative values for the relevant parameters. The collapse "mechanism" in those accounts is different from what has been proposed here, but a comparison can still be made in terms of the typical size that an entangled system can reach before superposition breaks down.

The dynamical reduction proposals employ a parameter, λ , which describes the frequency with which a typical one-particle wave function collapses. For purposes of comparison, we can use the value quoted by Pearle[50]²¹ of 10^{-16} per second. To achieve collapse in a relatively small fraction of a second requires an entangled system consisting of about 10^{18} (or more) elementary particles. This fairly conservative estimate is chosen, in part, to minimize the deviations from linear evolution, since no violations of the superposition principle have been observed at the level of elementary interactions.

For the hypothesis outlined in Section 3, consistency with current experimental results confirming superposition is maintained to a very high degree by the standard decoherence mechanism[47, 59]. The correlating interactions that are assumed to be responsible for nonlocal collapse effects make it difficult to observe any deviations from perfect linearity. Since the possible deviations tend to be hidden, it is conceivable that the collapse scale is much closer to the level of elementary interactions than is assumed in most dynamical reduction models. (The next section will explore ways to get around this "masking" effect, in order to derive feasible experimental predictions from the proposals made here.)

In fact, the general line of argument used in developing this proposal suggests a way to estimate the scale of wave function collapse. Information can only be instantiated in mesoscale and larger systems because of the probabilistic nature of elementary correlating interactions. This means that there is a lower bound on the

²¹Attributed to GRW[56]

size of naturally occurring information processing systems. We do not need to define the characteristics of these systems in precise detail in order to look at some examples of them.

So let us examine some very small biological systems. Given the span of time over which they have been evolving, it is reasonable to suppose that nature has explored the inherent limits involved in the construction of these entities. We will consider only those systems that are capable of operating independently within their environments, and carrying out all biological functions, including reproduction. This reasonably conservative assumption excludes viruses.

These organisms must be capable of exchanging energy with their environment to develop and maintain their internal organization. Some of their interactions, such as absorbing photons, involve probabilistic quantum processes. To enhance their prospects for survival and reproduction, it is plausible to suppose that these systems must be capable of independently resolving these processes. In other words, they must be able, independently, to completely collapse the wave function of the particles with which they interact.

In the previous section I hypothesized that the collapse process has the character of a random walk which terminates when the density of one of the branches reaches either 0 or 1. If the bifurcation defined by the interactions remains the same throughout the process, then this takes about $(1/\overline{d})^2$ steps, where \overline{d} is the average step size for the type of interaction involved. So organisms capable of inducing collapse, and remaining viable against the probabilistic outcomes of the collapse process must consist of somewhat more than $(1/\overline{d})^2$ elementary systems. It seems reasonable to suppose that the smallest such successful organisms must be on the order of 10 to 100 times the size of a system minimally capable of bringing about collapse.

The smallest microbes have linear dimensions of a few hundred nanometers [60]. This sets a rough size limit of about 10^{10} to 10^{11} elementary particles. If the organism is 10 to 100 times the size of an average collapse mechanism, we get an upper limit for an average wave function collapse of about 10^8 to 10^{10} elementary interactions. I will take the geometric mean of these numbers, 10^9 , as a nominal value for subsequent discussion.

This number represents the typical number of steps in the random walk that leads to complete wave function collapse. The average density shift, \overline{d} , should be roughly equal to the inverse of the square root of this number. So let us tentatively take $\overline{d} = 3 * 10^{-5}$, as our estimate of the average shift involved in ordinary elementary interactions. This will be used in the next section which outlines some experimental approaches to determine whether, and at what scale collapse occurs.

5 Experimental Consequences

Wave function collapse is not consistent with strictly linear Schrödinger evolution. Collapse entails the elimination of superposition effects at some stage of the measurement process. However, for a long time after the development of quantum theory, there was a tendency to dismiss or ignore the possibility of determining at what stage of the process the deviation from linear evolution occurs. This tendency was due, in large part, to the influence of Bohr[61] and Heisenberg[62, 63], who argued that it makes no difference where one draws the line between the measured system and the measuring apparatus.

The plausibility of the Bohr-Heisenberg argument is based on the difficulty of observing superposition effects in a system that has undergone interactions. The wave functions of single particles that have not interacted can be recombined fairly easily, and superposition can be exhibited through straightforward interference phenomena, as in simple double-slit or Stern-Gerlach experiments. If interactions occur, the branches of the wave function separate more widely in configuration space. Recombining the branches requires detailed control of all of the particles represented by the entangled wave function. Such detailed control is hard to achieve, so in most cases, the particles undergo further interactions. Branches of the wave function separate to an even greater extent, and any practical possibility of restoring coherence is lost[47]. In such a chain of interactions, the boundary between "system" and "apparatus" is clearly somewhat arbitrary.

Despite these difficulties, there are clear differences in experimental predictions between the assumption that the wave function collapses, and the view that unmodified linear evolution continues. These differences persist, in principle, to any level of entanglement generated by a chain of interactions. This point was emphasized by Bell[64].²²

The first practical tests of the persistence of superposition effects after interactions occur were proposed by Scully and his colleagues in the 1980's [66, 67, 68]. The key idea in these quantum eraser experiments is to separate the wave function of an elementary system into two (or more) branches, and have these branches (or at least one of them) interact with another "target" elementary system. One then recombines the branches of both subject and target, measures them in an alternate basis, and looks for correlations between them.

To illustrate the idea, consider an electron in a z-up state. If the x-up and x-down components are separated in a Stern-Gerlach device and then recombined without interacting with any other systems, then the superposition principle allows us to reconstruct the z-up state and detect it with a probability of 1.0 (assuming appropriate phases are maintained, with $\alpha = \beta = 1/\sqrt{2}$):

$$|z\uparrow\rangle \implies \alpha|x\uparrow\rangle + \beta|x\downarrow\rangle \implies |z\uparrow\rangle.$$
 (5.1)

²²For a somewhat different viewpoint, see Janssens and Maassen[65].

However, if at least one of the x-spin branches is allowed to interact with another elementary system prior to recombination, the possibility for simple interference is eliminated, and the probability of a z-up detection is reduced to 0.5: $(\gamma = \delta = 1/\sqrt{2})$

$$|z_1\uparrow\rangle \Rightarrow \alpha|x_1\uparrow\rangle + \beta|x_1\downarrow\rangle \implies \alpha|x_1\uparrow\rangle|x_2\uparrow\rangle + \beta|x_1\downarrow\rangle|x_2\downarrow\rangle = \gamma|z_1\uparrow\rangle|z_2\uparrow\rangle + \delta|z_1\downarrow\rangle|z_2\downarrow\rangle.$$
 (5.2)

(In the experiment proposed by Scully, Englert, and Schwinger[67] the role of the target is played by a pair of micromaser cavities. In these expressions $|x_2\uparrow\rangle, |x_2\downarrow\rangle$ correspond to micromaser number states, and $|z_2\uparrow\rangle, |z_2\downarrow\rangle$ represent micromaser symmetric/antisymmetric states.)

Despite the fact that we cannot reconstruct the z-up state in every case, superposition is still exhibited in the perfect correlations between detections of $|z_1\uparrow\rangle$ and $|z_2\uparrow\rangle$ (a z-up electron state and micromaser symmetric states), and also between $|z_1\downarrow\rangle$ and $|z_2\downarrow\rangle$. The dependence of the correlations on superposition can be shown explicitly by expanding the terms in equation 5.2 separately:

$$\alpha |x_1\uparrow\rangle |x_2\uparrow\rangle = \alpha (1/\sqrt{2})^2 (|z_1\uparrow\rangle + |z_1\downarrow\rangle) \otimes (|z_2\uparrow\rangle + |z_2\downarrow\rangle)$$

$$= \alpha (1/\sqrt{2})^2 (|z_1\uparrow\rangle |z_2\uparrow\rangle + |z_1\downarrow\rangle |z_2\downarrow\rangle + |z_1\uparrow\rangle |z_2\downarrow\rangle + |z_1\downarrow\rangle |z_2\uparrow\rangle)$$
(5.3)

and

$$\beta |x_1\downarrow\rangle |x_2\downarrow\rangle = \beta(1/\sqrt{2})^2(|z_1\uparrow\rangle - |z_1\downarrow\rangle) \otimes (|z_2\uparrow\rangle - |z_2\downarrow\rangle)$$

$$= \beta(1/\sqrt{2})^2(|z_1\uparrow\rangle |z_2\uparrow\rangle + |z_1\downarrow\rangle |z_2\downarrow\rangle - |z_1\uparrow\rangle |z_2\downarrow\rangle - |z_1\downarrow\rangle |z_2\uparrow\rangle)$$
(5.4)

The form in 5.2, which displays the perfect correlations, results because of the cancellation of the up-down cross terms, $(|z_1 \uparrow\rangle |z_2 \downarrow\rangle$ and $|z_1 \downarrow\rangle |z_2 \uparrow\rangle)$, that occurs when 5.3 and 5.4 are superposed (since $\alpha = \beta$).

We can see the clear inconsistency between the assumptions of continued linear evolution and projection by supposing that the interaction between the electron and the micromaser cavity had constituted a full measurement (in the sense of inducing complete collapse). In that case the coefficients, α and β , would have been changed to 1 and 0, or 0 and 1. The correlations would be totally eliminated.

This inconsistency is *not* removed by adding more interactions. To see this, consider an entangled system with many particles. Label the states of the "subject" and (multiple) "detector" systems as $|x_s\rangle, |z_s\rangle, |x_{di}\rangle, |z_{di}\rangle$. Designate the superposition of all normalized product states with an even number of $|z\downarrow\rangle$ detector states as $|z_d\downarrow_{EVEN}\rangle$, with $|z_d\downarrow_{ODD}\rangle$ representing the complementary states.²³ (Normalization factor = $(1/\sqrt{2})^{N-1}$.) After N correlating interactions in the x-spin basis, we get

$$\alpha |x_{s}\uparrow\rangle |x_{d1}\uparrow\rangle ... |x_{dN}\uparrow\rangle$$

$$= \alpha (1/\sqrt{2})^{N+1} [(|z_{s}\uparrow\rangle + |z_{s}\downarrow\rangle) \otimes (|z_{d1}\uparrow\rangle + |z_{d1}\downarrow\rangle) ... (|z_{dN}\uparrow\rangle + |z_{dN}\downarrow\rangle)]$$

$$= (\alpha/2) [|z_{s}\uparrow\rangle |z_{d}\downarrow_{EVEN}\rangle + |z_{s}\downarrow\rangle |z_{d}\downarrow_{ODD}\rangle + |z_{s}\uparrow\rangle |z_{d}\downarrow_{ODD}\rangle + |z_{s}\downarrow\rangle |z_{d}\downarrow_{EVEN}\rangle],$$
(5.5)

²³Even numbers include zero, which covers the case described by 5.2.

and

$$\beta |x_{s}\downarrow\rangle(|x_{d1}\downarrow\rangle...|x_{dN}\downarrow\rangle$$

$$= \beta(1/\sqrt{2})^{N+1}[(|z_{s}\uparrow\rangle - |z_{s}\downarrow\rangle) \otimes (|z_{d1}\uparrow\rangle - |z_{d1}\downarrow\rangle)...(|z_{dN}\uparrow\rangle - |z_{dN}\downarrow\rangle]$$

$$= (\beta/2)[|z_{s}\uparrow\rangle|z_{d}\downarrow_{EVEN}\rangle + |z_{s}\downarrow\rangle|z_{d}\downarrow_{ODD}\rangle - |z_{s}\uparrow\rangle|z_{d}\downarrow_{ODD}\rangle) - |z_{s}\downarrow\rangle|z_{d}\downarrow_{EVEN}\rangle].$$
(5.6)

If no collapse occurs after N correlating interactions, then $\alpha = \beta$, and superposition of the two expressions yields cancellation of the terms, $|z_s \uparrow\rangle |z_d \downarrow_{ODD}\rangle$) and $|z_s \downarrow\rangle |z_d \downarrow_{EVEN}\rangle$. This gives a perfect correlation between up results of z-spin measurements on the subject particle, and an *even* number of down results from z-spin measurements on the detector particles (with the opposite correlation for down results on the subject particle).²⁴ If, on the other hand, α and β have been changed to 1 and 0, or 0 and 1 by a collapse, then there is no cancellation of the cross terms, and the correlations are eliminated.²⁵

It should be noted that the strategy of inducing particles to interact in one basis, and measuring them in a different basis constitutes the essential "trick" of quantum computing, as emphasized by Mermin[71]. So efforts to construct quantum computers also serve as tests of whether and when collapse occurs. Most of the work in quantum computing proceeds on the assumption that one is free to set the boundary between linear Schrödinger evolution and measurement at any convenient point (aside from pragmatic concerns about decoherence). However, the (assumed) linear evolution typically includes correlating interactions of the same general sort as are involved in measurement processes. If wave function collapse is a real physical process associated with these interactions, then it will set in-principle limits on how far out one can place the measurement boundary.

Given these considerations, we can summarize the status of current experimental results. Expected superposition effects have been observed in double-slit quantum eraser experiments[52], and quantum computations involving a few elementary interactions have been successfully implemented[53, 54, 55]. These results indicate that any deviations from perfectly linear evolution in individual interactions are small compared to the typical amplitudes of the particles involved.

To assess the feasibility of detecting the deviations that were hypothesized in Section 3, let us take another look at the situation represented by equations 5.5 and 5.6 above. As stated, if no deviations from linearity have occurred after N interactions, then $\alpha = \beta$, and the perfect correlation between z-up results on the subject particle, and an even number of down results on the detector particles is maintained. If, on

²⁴Of course, the detector states can be any appropriate correlated states (such as micromaser states); the z-spin designation is just for convenience.

 $^{^{25}}$ The question of whether and when deviations of this general nature occur must be faced by any account of the measurement problem. No-collapse interpretations such as that of Everett[69], decoherence accounts[70], and de Broglie-Bohm theory[29, 30] (with "correct" initial distributions) predict zero deviations from the perfect correlations no matter how large N is. Spontaneous collapse theories[56, 57, 50, 58] imply deviations of some magnitude at some stage of the measurement process.

the other hand, density shifts associated with the interactions have changed α and β to α' and β' with $\alpha' \neq \beta'$ then we do not get (complete) cancellation, and the correlations are altered.

If we use the estimate of the average step size, d, from the previous section, we can make a provisional calculation of the size of the deviations predicted by the shift hypothesis for this particular situation. In a random walk of N steps the average deviation from the starting point is \sqrt{N} . The initial densities are the squared amplitudes, $\alpha\alpha^*$ and $\beta\beta^*$. After N interactions these would be altered on average to $\alpha'\alpha'^* = \alpha\alpha^* \pm (\sqrt{N})d$, and $\beta'\beta'^* = \beta\beta^* \mp (\sqrt{N})d$. For values of $(\sqrt{N})d \ll \alpha\alpha^*$, $\beta\beta^*$, these differences imply changes in the amplitudes of approximately $(\sqrt{N})(d/2\alpha)$ and $(\sqrt{N})(d/2\beta)$. By substituting the values $(\alpha\pm d\sqrt{N}/2\alpha)$ and $(\beta\mp d\sqrt{N}/2\beta)$ for α and β in expressions 5.5 and 5.6 and superposing them, we can compute the amplitude coefficients of the cross terms, $(|z_s\uparrow\rangle|z_d\downarrow_{ODD}\rangle)$ and $(|z_s\downarrow\rangle|z_d\downarrow_{EVEN}\rangle)$. If the initial values of α and β are $1/\sqrt{2}$, then the amplitude for the cross terms is approximately $(\sqrt{N/2})d$. So the probability of detecting these "deviant" cases is about $(N/2)d^2$. The estimate for d^2 from Section 4 was of order 10^{-9} . It would obviously be quite difficult to detect such deviations in these kinds of experimental arrangements.

To increase the likelihood of observing possible departures from linear evolution, we need to maximize the deviations in the *amplitudes* of the components. Our working assumption is that deviations in the *densities* will be roughly the same size in most cases.²⁶ The small size of the value derived above, $(\sqrt{N/2})d$, results largely from the condition, $(\sqrt{N})d \ll \alpha\alpha^*$, $\beta\beta^*$. To increase the size of the possible amplitude shifts, we can work with states in which $d \approx \alpha\alpha^*$. This will give us amplitude differences of order \sqrt{d} , instead of d.

We can calculate the probability of a deviation from linear evolution in terms of d as follows. As before, we begin with a state, $\alpha |x_s\uparrow\rangle + \beta |x_s\downarrow\rangle$, but now with $\alpha \ll \beta$. Entanglement is generated by inducing one of the x-spin branches to interact with a detector system; we then look for correlations with z-spin branches:

$$(\alpha|x_s\uparrow\rangle + \beta|x_s\downarrow\rangle) \otimes |x_d - ready\rangle \Longrightarrow \alpha|x_s\uparrow\rangle|x_d\uparrow\rangle + \beta|x_s\downarrow\rangle|x_d\downarrow\rangle$$

If $\alpha \ll \beta$, the resulting state can still be represented in the form:

$$(1/\sqrt{2})|z_s\uparrow\rangle|u_d\rangle + (1/\sqrt{2})|z_s\downarrow\rangle|v_d\rangle,$$

but now the correlates of $|z_s\uparrow\rangle$ and $|z_s\downarrow\rangle$ are not orthogonal. Assuming no collapse effects they can be easily calculated:

$$|u_d\rangle = (\alpha + \beta)(1/\sqrt{2})|z_d\uparrow\rangle + (\alpha - \beta)(1/\sqrt{2})|z_d\downarrow\rangle,$$

$$|v_d\rangle = (\alpha - \beta)(1/\sqrt{2})|z_d\uparrow\rangle + (\alpha + \beta)(1/\sqrt{2})|z_d\downarrow\rangle.$$

²⁶The reason for framing the hypothesis in terms of density shifts instead of amplitude shifts was to maintain strict adherence to the Born Rule, and thus preserve causality.

With no deviation from linearity one should still observe perfect correlations between $|z_s\uparrow\rangle$ and $|u_d\rangle$ and between $|z_s\downarrow\rangle$ and $|v_d\rangle$. However, if there is a shift, d, in the density due to the interaction, then α and β are shifted to α' and β' , and $|u_d\rangle$ and $|v_d\rangle$ are shifted to

$$|u'_{d}\rangle = (\alpha' + \beta')(1/\sqrt{2})|z_{d}\uparrow\rangle + (\alpha' - \beta')(1/\sqrt{2})|z_{d}\downarrow\rangle,$$

$$|v'_{d}\rangle = (\alpha' - \beta')(1/\sqrt{2})|z_{d}\uparrow\rangle + (\alpha' + \beta')(1/\sqrt{2})|z_{d}\downarrow\rangle.$$

The effect on the correlations between the $|z_s\rangle$ and original $|u_d\rangle$ and $|v_d\rangle$ states can be computed by taking the inner product between the original and primed states. With α and β real (for simplicity) we get

$$\langle u'_d | u_d \rangle = (1/2)(\alpha \alpha' + \beta \beta' + \alpha \beta' + \beta \alpha') + (\alpha \alpha' + \beta \beta' - \alpha \beta' - \beta \alpha')$$

= $(\alpha \alpha' + \beta \beta')$

$$\langle v'_d | v_d \rangle = (1/2)(\alpha \alpha' + \beta \beta' - \alpha \beta' - \beta \alpha') + (\alpha \alpha' + \beta \beta' + \alpha \beta' + \beta \alpha')$$

= $(\alpha \alpha' + \beta \beta')$

Collapse effects (density shifts) are indicated by deviations from the perfect correlations predicted by strictly linear evolution. Their magnitude is:

$$1 - (\alpha \alpha' + \beta \beta')^2 = 1 - (\alpha \alpha \alpha' \alpha' + \beta \beta \beta' \beta' + 2\alpha \alpha' \beta \beta').$$

Suppose that $\alpha\alpha = d$. The density shift hypothesis implies that in half the cases $\alpha\alpha$ would be shifted to $\alpha'\alpha' = 0$, and $\beta\beta$ would be shifted to $\beta'\beta' = 1$. In these situations we get $1 - \beta\beta = \alpha\alpha$; the statistical deviation for this half of the cases will be $\alpha\alpha = d$. In the other half of the cases $\alpha\alpha$ would be increased to $\alpha\alpha + d = 2\alpha\alpha$, and $\beta\beta$ would be decreased to $\beta\beta - d$. With $\beta\beta = 1 - \alpha\alpha$ and $\beta'\beta' = 1 - 2\alpha\alpha$, $\beta \approx 1 - \alpha\alpha/2$, $\alpha' = \sqrt{2}\alpha$, we get

$$1 - [(1 - \alpha\alpha)(1 - 2\alpha\alpha) + 2\alpha\alpha\alpha\alpha + 2\sqrt{2}\alpha\alpha(1 - \alpha\alpha/2)(1 - \alpha\alpha)]$$
$$= 3\alpha\alpha - 2.8\alpha\alpha - (higher\ order\ terms) \approx 0.2\alpha\alpha.$$

So in half the cases the deviation is d, and in the other half it is about 0.2d, for an average of about 0.6d. The estimate for d is about $3*10^{-5}$ (approximately 1 part in 30,000), so it might be experimentally accessible. Of course, with α being small there would be a need to control and measure the initial state very precisely.

Although experimental tests of wave function collapse face substantial challenges, the efforts being made to construct quantum computers give some hope that such tests can be made in the not too distant future. If quantum computers reach the stage where the entangled systems involved in the computations include millions of elementary particles, the possible deviations from perfect linearity could become noticeable.

In closing this section it is important to emphasize that the deviations from linear evolution predicted by the density shift hypothesis do not violate causality. To observe any deviations (if they exist), one must recombine branches of the wave function, and measure them in a basis other than the one in which they have interacted. This implies that the final measurements must be timelike later than those interactions that established the initial entanglement relations, so the possibility of superluminal signaling never arises.

6 Discussion

The argument of the previous section shows that there are clear inconsistencies in the experimental predictions of quantum theory depending on where one draws the line between measured system and measuring apparatus. This problem stems from the clash between the deterministic, (mostly) local evolution described by the dynamic equations governing elementary processes, and the probabilistic, nonlocal nature of the measurement postulates. These inconsistencies can be obscured by the size and complexity of measurement arrangements, but they are always there, in principle. As experimental techniques advance, and quantum computational apparatus becomes more sophisticated, the potential conflicts will become even sharper.

To resolve the discrepancies, we must carefully examine the conceptual framework of contemporary physics. The nonlocal and nondeterministic effects described by the measurement postulates resist easy incorporation into a relativistic spacetime structure. Nevertheless, the standard version of quantum measurement does share at least one key property with conventional interpretations of relativity based on local deterministic theories. Conventional relativity and standard measurement theory both prohibit the superluminal transmission of information. Since signal causality is the most obvious feature that these two apparently disparate aspects of current theory have in common, it makes sense to take this principle as a starting point in any attempt to reconcile them.

As stated in the introduction, Bell criticized this approach because it can be both imprecise and subjective. His arguments have merit, but the proper response should not be to abandon the only readily identifiable feature shared by the "two fundamental pillars of contemporary theory" [72]. Instead, we should try to fix the shortcomings.

We can eliminate the problem of subjectivity by noting the connection of causality to the concept of information, and insisting that this concept be definable in purely physical terms. We do not need to develop a complete account of what it means for physical systems to instantiate and transmit information, but we do need to be sure that any inferences that we draw based on the concept are consistent with an interpretation of information as a strictly physical relationship.

When we take this approach, we immediately see some very specific consequences of signal causality. Given the nonlocality of projection effects, one can immediately derive the Born Probability Rule, as demonstrated in Gleason's theorem. It also follows that any physical process capable of observing nonlocal effects must be capable of inducing those effects, so projection must be tied to measurementlike processes. By extending this inference to the microphysical level, we see that the probabilistic, nonlocal effects must stem from the elementary interactions that generate entanglement.

So, the limits on determinism that are implied by these considerations reconcile the existence of real nonlocal effects with the impossibility of using them to transmit information. Because the interactions that constitute measurements are not deterministic, information can only be instantiated in systems that involve enough correlating interactions and replicating subsystems to fully resolve probabilistic processes and insure reasonably stable records of the outcomes. When states are changed by spacelike-separated interactions, the local physical record of the previous state is eliminated, unless it is stored in a system capable of recording, interrogating, and comparing the states of a sufficient number of subsystems. These types of processes are of the sort that bring about wave function collapse. So there is never any physical representation that a collapse has been brought about by a spacelike-separated set of interactions.

The specific consequences and general insights that follow from the principle of signal causality, properly interpreted, should give us some confidence that it can be further exploited to help bridge the gap between relativity and quantum nonlocality.

To resolve the conflict, we must first recognize how sharp it is. Any coherent explanation of wave function collapse at the level of elementary interactions must deal with two related types of nonlocal effects: (1) the familiar ones that project out one particular branch of an entangled state, and (2) the generation of the entanglement relations that mediate the collapse and define the subsystems (and states) that are being acted on. In any realistic collapse process there are huge numbers of spacelike-separated elementary interactions that are involved in myriad nonlocal effects. A consistent microphysical description of projection must be able to deal with this complexity.

We can construct a very straightforward account of probabilistic, nonlocal effects by assuming that there is *some sequence* to those effects. With such an assumption, the state on which those effects act is well defined, and the need for cooperation among the spacelike-separated processes that generate the effects is eliminated. Any attempt to describe collapse at the elementary level without such a sequencing assumption faces extreme difficulties.

Of course, any objective sequencing of spacelike-separated events is at odds with the idea that spacetime structure is completely described by relativity. To deal with this dilemma, consider the intimate connection between causality and relativity. We have seen that the key to preserving causality in the face of nonlocal measurement effects is the nondeterministic nature of those effects. If we assume that these two defining properties stem from the same roots, then that nondeterministic nature should also explain the emergence of relativity.

The indeterminism of nonlocal collapse effects preserves signal causality not by negating the reality of those effects, but rather by requiring any observational process to be of macroscopic (or at least mesoscopic) scale. The same indeterminism preserves relativity not by denying the objective existence of sequences of spacelike-separated interactions, but rather by insuring that those sequences lie below the threshold of observability. So relativity is not a complete characterization of spacetime structure. It is a property of the family of empirically equivalent descriptions of the processes that occur in spacetime. The lack of determinism in elementary interactions means that objectively distinct sequences of elementary events correspond to the same set of observations. This is what guarantees the equivalence of all empirically definable reference frames.

The overarching lesson is that Bell's analysis has shown that we cannot unify physics by explaining macroscopic measurement results in local deterministic terms. Instead we must find a way to incorporate the nonlocal, nondeterministic aspects of reality into our description of elementary processes. We can do this by recognizing the following key points. Signal causality implies that nonlocal measurement effects originate in the elementary interactions that generate entanglement, and that these effects are essentially probabilistic. A coherent account of these effects requires that spacelike-separated interactions involved in a collapse process have some objective sequence. The indeterminism that protects signal causality also maintains relativity by making these sequences unobservable, in principle.

Once these points are granted, it is fairly easy to construct a phenomenological description of collapse at the microphysical level. With every elementary interaction that generates entanglement we associate a probabilistic shift of density (squared amplitude) either into or out of the interacting branch. The binary decomposition of the wave function into interacting and noninteracting branches provides a natural definition of what is being measured. The Projection Postulate and the Born Probability Rule are readily reproduced. Together with the recognition that information can only be instantiated at mesoscopic and larger scales, this maintains signal causality, and preserves the relativistic transformation properties of empirically equivalent descriptions of events in spacetime.

In closing, it is necessary to point out some limitations of the ideas presented here. First, this account is just a phenomenological outline of a solution to the problem. If it is on the right track, it should eventually be supplanted by a more thorough mathematical treatment that would show how the standard quantum description in relativistic spacetime can be recovered by averaging over the possible sequences of probabilistic nonlocal effects. At a deeper level, there remains a sort of dualism in the picture of physical processes and spacetime. Most deterministic processes are still confined to the light cone, while certain types of nondeterministic effects can propagate across spacelike intervals. The concept of signal causality summarizes what is common to these two aspects of physical reality, but it does not provide a

clear vision of how these apparently distinct manifestations emerge from a unifying principle.

Nevertheless, we can move toward a more unified view by better understanding the connection between macroscopic and microscopic realms. To do this we need to integrate the measurement postulates with our theories of elementary interactions by taking seriously the nonlocality and the limits on determinism implied by those postulates.

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